

# DEVELOPING CRITICAL THINKING SKILLS IN THE TEACHING AND LEARNING OF MATHEMATICS

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## Abstract

Mathematics as a science of skillful operations possesses the attribute required for the training of the mind to think. Thus, the study explored the definition of Mathematics, thinking and thinking skills, critical thinking skills, importance of critical thinking skills appropriate standards to the assessment of critical thinking skills. It also x-rays the critical thinking process, the importance of critical thinking skills, characteristics of critical thinking skills, teacher's role in developing critical thinking skills, assessment qualities for how well students can think, approaches to fostering critical thinking. Specifically, the study established the instructional procedure for developing critical thinking skills using the concept of Pie(π). It was recommended that teachers should stop teaching calculations in mathematics lessons, rather they should start developing critical thinking skills in the learners during mathematics lessons. This will help the learners to understand the links between ideas, determine the importance and relevance of arguments and ideas, recognize, build and appraise arguments.

## INTRODUCTION

Mathematics as an old broad and deep field of study, people working to improve mathematics education need to understand what is mathematics? Mathematics is an interdisciplinary language and tool which is considered basic in our formal education system. Eugene Wigner in his own contribution defines mathematics as the science of skillful operations of concepts and rules invented just for this purpose. The skillful operations can only be realized if the citizenry can think critically.

According to Albert Einstein "Education is not learning of facts, but training the mind to think". Throughout history philosopher, politicians, educators and many others have been concerned with the art of science of astute of thinking. Some identified the spirit of inquiry and dialogue that characterized the golden age of ancient Greece's as the beginning of this interest. Others points to the Age of Enlightenment, with its emphases on rationality and program (Presseien in Ndirika and Agommuoh,2014) In the

twentieth century, the ability to engage in carefully, reflective thought has been viewed in various ways: as a fundamental characteristic of an educated person; as a requirement for responsible citizenship in a democratic society; and more recently, as an employability skill for an increasingly wide range of jobs .Perhaps, most importantly in today's information age, thinking skills are viewed as crucial for an educated person to cope with a rapidly changing world. Many educators believe that specific knowledge will not be as important to tomorrows workers and citizen as the ability to learn and make sense of new information. Educators are not alone in recognizing the importance of critical thinking because the demands of employment in a global economy, the survival of a democratic way of life, and personal decision making in a complex and rapidly changing society, require people who can reason well and make good judgments. As African are desiring to move toward a technology-based economy so as to be equipped to face world-wide competition, employers

demand workers who can think flexibly and analytically, integrate information from a variety of sources and perspectives, and make profitable decisions efficiently.

For students, workers, and citizens, critical thinking is an essential tool for performing successfully in a complex and rapidly changing world. In each of these roles, as David Perkins (1989) points out, we must examine the factors impinging on a situation, forecast the outcomes of possible courses of action, evaluate those outcomes and weigh them relative to one another, and try to choose so as to maximize positive outcomes and minimize negative ones. Further, the beliefs we hold, and consequently the inferences we later make and attitudes we later assume, depend in part on our reasoning about the grounds for those beliefs. Accepting beliefs wisely serves the ultimate end of later sound conduct as well as the more immediate end of sound belief itself.

Despite widespread expressions of concern about developing critical thinkers, studies have shown that most schools are neither challenging students to think critically about academic subjects nor helping them develop the reasoning abilities needed to deal successfully with the complexities of modern life. Our educational system continues to graduate students who do not reason well (Goodlad, Keating, Kennedy, & Paul in Beam Evelyn & Markhan Houston (2008).

### **THINKING AND THINKING SKILLS**

Thinking is a complex process, and its specific deviations or parts, are not distinct. Each increasing level of the

hierarchy of thinking makes use of the skills contained in lower levels. The art of thinking requires interaction among all the levels of cognition as recalling information which is a skill that is almost automatic in nature. For each learner, the recall block is different. As children continue to make association in early life the recall block expand to accommodate this increase in information, for some primary pupils the recall block might include basic addition facts or the understanding of letter sound relationships yet there can be same age children who have not committed there facts to their recall memory block, and therefore cannot recall up-this information as needed.

Basic thinking includes the understanding of simple mathematical concepts such as addition and subtraction, and decoding print. Basic thinking also includes task such as looking up vocabulary in a glossary, single criterion classification. Applying these skills in everyday situation in and outside of school is also a basic thinking function. Thinking skill can be defined as the conservational abilities for an individual to reason or hypothesize abstractly, thereby, build up new mental restructure through the construction of new operation which can be logically retained and applied wherever necessary. Thinking skills require finding a solution which is unique and novel to an identified mathematical problem. It calls for visualizing, imagining, monitoring, analyzing, abstracting and synthesizing of ideas cognitively or mentally or insightfully. The responses are often unique novel, original, and varied from one problem to another.

### **WHAT IS CRITICAL THINKING?**

History and background of critical thinking can be traced to the teaching, practice, and vision of Socrates, who established the importance of asking probing questions rather than just accepting ideas as worthy of belief. He set the basic constructs of critical thinking, including such actions as reflectively questioning common beliefs and explanations and carefully distinguishing those beliefs that are reasonable and logical from those that lack adequate evidence or a rational foundation

Critical thinking is the mode of thinking about any subject ,content ,or problem in which the thinker improves the quality of his or her thinking by skillfully taking charge of the structure inherent in the thinking and imposing intellectual standards upon them. Critical Thinking is complex of skills and dispositions which is the potential ,natural tendencies or personal inclusions to demonstrate critical thinking skills .There are seven critical thinking dispositions .They are **t r u t h s e e k i n g , o p e n - m i n d e d n e s s , a n a l y t i c a l , s y s t e m a t i c i t y , s e l f - c o n f i d e n c e , i n q u i s i t i v e n e s s** and cognitive maturity (Paul R.&Elder L.,2008)

Pauls model for critical thinking skills insist that the universal standard criteria for critical thinking by which all, who attempts to think critically should be measured are clarity, accuracy, precision,relevance, consistency, depth and breadth. How well a student is reasoning depends on how well he/she applies these universal standards to the elements (or parts) of thinking.

Beam Evelyn & Markhan Houston (2008) defines critical thinking as being able to think rationally, independently, strategically, judiciously, reflectively, inferentially and creatively, in order to make informed decision address issues answer complex questions an solve problem confronted by instructions. Critical thinking can be described as an informal logic, or sometime referred to as the study of reasoning and fallacies in the context of everyday life. No wonder, Paul,R. and Elder L. (2008) suggested two essential dimensions of thinking that students need to master in order to develop as fair-mindedness as a critical thinkers. They need to be able to identify the "parts" of thinking, and they need to be able to assess use of these parts of thinking, as follows

1. All reasoning has a **PURPOSE:**
  - Take time to state your purpose clearly
  - Distinguish your purpose from related purposes
  - Check periodically to be sure you are still on target
  - Choose significant and realistic purposes
2. All reasoning is an attempt to **FIGURE SOMETHING OUT, TO SETTLE SOME QUESTION, TO SOLVE SOME PROBLEM:**
  - Take time to clearly and precisely state the question at issue
  - Express the question in several ways to clarify its meaning and scope

- Break the question into sub questions
  - Identify if the question has one right answer, is a matter of opinion, or requires reasoning from more than one point of view
3. All reasoning is based on **ASSUMPTIONS**:
- Clearly identify your assumptions and determine whether they are justifiable
  - Consider how your assumptions are shaping your point of view
4. All reasoning is done from some **POINT OF VIEW**:
- Identify your point of view
  - Seek other points of view and identify their strengths as well as weaknesses
  - Strive to be fair-minded in evaluating all points of view
5. All reasoning is based on **DATA, INFORMATION and EVIDENCE**:
- Restrict your claims to those supported by the data you have
  - Search for information that opposes your position as well as information that supports it
  - Make sure that all information used is clear, accurate, and relevant to the question at issue
  - Make sure you have gathered sufficient information
6. All reasoning is expressed through, and shaped by, **CONCEPTS and IDEAS**:
- Identify key concepts and explain them clearly
  - Consider alternative concepts or alternative definitions to concepts
  - Make sure you are using concepts with care and precision
7. All reasoning contains **INFERENCE**s or **INTERPRETATIONS** by which we draw **CONCLUSIONS** and give meaning to data:
- Infer only what the evidence implies
  - Check inferences for their consistency with each other
  - Identify assumptions which lead you to your inferences
8. All reasoning leads somewhere or has **IMPLICATIONS** and **CONSEQUENCES**:
- Trace the implications and consequences that follow from your reasoning
  - Search for negative as well as positive implications
  - Consider all possible consequences

### THE CRITICAL THINKING PROCESS

This is everyday life process which individuals who uses critical thinking skills (CTS) will be able to apply in a given educational and learning situation. There are five steps involved in the critical thinking process namely:

- Step 1: Analyze and gather relevant data about the problem or situation.  
 Step 2: Select plausible solutions'/conclusion.  
 Step 3: implement the best solution  
 Step 4: Evaluation or track the effect or impact of the selected solution.  
 Step 5: Revise/change solution of best result
- Some people believe that critical thinking is different from creativity because critical thinking follows the rules of logic and rationality while creativity might at some stages break these rules in order to present desired outcome.

### THE IMPORTANCE OF CRITICAL THINKING SKILLS.

- the ability to think clearly and rationally is important to whatever we choose to do.
- The ability to think well and solve problems systematically is an asset for any career.
- critical thinking is very important in the global knowledge economy which is driven by information and technology that culminates in fast changing workplace.
- critical thinking enhance language and presentation skills: Thinking clearly and systematically can improve the way we express our ideas and improves comprehension abilities.
- critical thinking promotes creativity: To come up with a creative solution to a problem situation, it is necessary that new ideas generated are relevant toward solving the problem. At this point, critical thinking helps to evaluate the new ideas by selecting the best ones and as well modify them if necessary.
- critical thinking is crucial for

self- reflection : critical thinking as a tool will help us to maintain and leave a meaningfully and structured way of life which will justify and reflect our values and decisions.

- critical thinking is the foundation of science and democracy: science require the critical use of reason in experimentation and theory confirmation. The proper functioning of a liberal democracy require citizen who can think critically about social issues to inform their judgment about proper governance and to overcome biases an prejudice.
- critical thinking helps you to better understand the experience and ways of others, that is, it enhances our ability to work with different people.
- critical thinking helps students do better especially at the mathematical reasoning test.
- critical thinking helps students to become more independent and a self- directed learner rather than relating on teachers and classroom time for instruction and guidance.

### CHARACTERISTICS OF CRITICAL THINKING SKILLS

- Ø Understanding of the logical connection between ideas.
- Ø Identify, construct and evaluate arguments.
- Ø Detect inconsistencies and common mistakes in reasoning
- Ø Solve problems systematically
- Ø Identify the relevance and importance of ideas.
- Ø Reflect on the justification of one's own beliefs and values

## TEACHER'S ROLE IN DEVELOPING CRITICAL THINKING SKILLS

The role expected of a teacher in developing critical thinking is different from the role that is played in a conventional mathematics classroom. To engaged students in critical thinking, the teacher need to

- Act as a facilitators to allow for discussion and encourage a freer thought process.
- Encourage understanding that thinking critically does not always end with a right answer but instead sometimes ends in more questions of differing evaluation of the topic (Halx Reybold 2005 & Aren, 2009).
- Access how well their students can think?

## ASSESSMENT QUALITIES ON HOW WELL STUDENTS CAN THINK

A Mathematics teacher can assess how well the students think by assessing how they

- value objective reality
- keep an open mind
- do not tolerate ongoing and unproductive ambiguity
- avoid the bandwagon
- distinguish between observation and inference, between established facts and conjectures that follow
- withhold judgement until you are sure you have adequate information
- maintain a sense of humor
- cultivate intellectual curiosity
- don't take things on face value
- challenge conventional wisdom
- resist appeals to motion
- do not automatically accept authority
- beware of your own ego enhancing

- behavior
- maintain a sense of perspective
- beware of unspoken rules
- beware of non verbal behavior clues
- stop and think when under pressure
- see beyond labels and stereotype
- weed out negative self talk
- look for consistency
- practice empathy
- take time to check the facts
- check the validity of your information

## APPROACHES TO FOSTERING CRITICAL THINKING

- There are four main approaches to foster critical thinking among students are: inquiry; questioning; problem solving; and collaboration.
- Inquiry: The teacher allows the students to enquire about topics thereby searching for new information, demonstrate an understanding and share their new learning with others.
- Questioning: The students are guided by questions which will help them to dig deeper in the learning situation. The teacher as well assists them by providing a variety of resources to help them to independently find answers to the problem.
- Problems solving: The teacher do not give students answer to the problem at hand rather the problem are turn onto the students to suggests ways on how to solve the problem.
- Collaboration: The students are actively involved into the leaning situation by collaborative work. This which will help the students take ownership of the learning and think critically about issue

## INSTRUCTIONAL PROCEDURE FOR DEVELOPING CRITICAL THINKING SKILLS

### Topic: the concept of Pie( $\pi$ )?

Teaching Method: the teacher facilitates the students towards the realization of the expected response

**Knowledge:** This is the cognitive stage of remembering previously materials. At this stage teachers should endeavor to ask the right questions by using some words and phrases such as: name, recall, how many, when, where, list, define, tell, describe, identify, who is,..... What is,..... When did

- **Expected sample question:** What is pie ( $\pi$ )
- **Expected response :** Pie is  $\frac{22}{7}$

**Comprehension:** this is the stage of understanding the meaning of remembered materials. At this stage some words such as: describe, predict identify differentials review, explain, estimate the sample question at this category could be used.

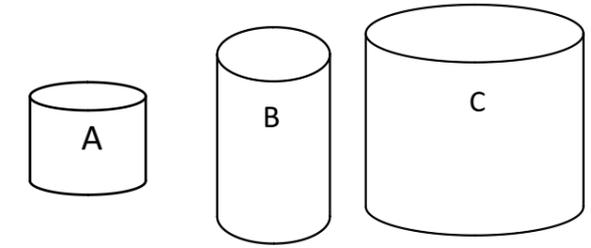
- **Expected sample question :** Explain the concept of pie.

**Expected response:** Pie is the ratio of the circumference of a circular region to its diameter. It has a constant 3.14(2 decimal places or 3 significant figures) or  $\frac{22}{7}$

**Application:** This is the stage whereby the information acquired at the (knowledge and comprehension) are used in a new way. At this stage teacher should ask questions rightly by using some words and phrases like demonstrate, apply, reinstate, show, solve, examine.

**Expected sample question:** show that pie is a constant equal 3.14(2 decimal places or 3 significant figures) or  $\frac{22}{7}$ .

using the three sizes of tin tomatoes labeled A,B,C as shown below



- **Expected response:** the circumferences and the diameter of the circular top of the three tins were measured using thread or ropes and metre rule .
- The ratio  $\frac{\text{circumference}}{\text{Diameter}}$  -is calculated for each ,

**Analysis:** This stage involves breaking down information into parts, and or examining the parts. At this stage valid decisions are drawn from the application stage,by the teacher asking the students the following questions: what did you observe? What can you infer from the result? What conclusion can we make?

- **Expected sample question :**What do you observe in the three measurements and ratio calculation
- **Expected response:** the circumferences and diameter increases as the sizes of the cylindrical tins; the result of the computation of ratios are all approximately equal to 3.14 to 3 significant figures

**Synthesis:** This stage involves bringing ideas or sets of abstract relations together to form a new one. At this stage teacher should ask questions rightly by using some words like produce, develop, compose, design, create, rearrange, reconstruct, plan, produce, formulate etc.

-**Expected sample question:** form a cone with the same circumference and diameter as any of the given cylinders

-**Expected response:** the students

will observe that the net of a cone is the sector of a circle and as such should have the concept of Pie in its formula

EVALUATION: This stage is the highest level of cognition where the teacher examines an issue to determine whether it is worthwhile or not. At this stage teacher should ask questions rightly by using some words like justify, argue, support, appraise, validate, criticize e.t.c

**Expected sample question: justify by formula method the concept of pie**

**-Expected response:**

$$\text{pie } (\pi) ? = \frac{\text{circumference}}{\text{Diameter}} = \frac{2\pi r}{2r}$$

**Conclusion:** Most mathematics teachers agree that thinking skills can be acquired through adequate practice, intensive repetition and through drilling using various procedures. But nowadays, teachers often repeat exercises using the same procedures for solution, which may lead to boredom and development of apathy for individual thinking skills. Also teachers always asking question with a little or no practice, this questions can be used to help the learner develop their critical thinking skills. The development of critical thinking skills during mathematics lessons will help the learners to understand the links between ideas, determine the importance and relevance of arguments and ideas, recognize, build and appraise arguments.

#### **Recommendation**

To maximize the benefit of this study towards national development, the mathematics teacher should stop teaching calculations, rather they should engage the students in a rigorous process of developing the critical thinking skills in them during mathematics lessons

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